

Chaos and Quantum Chaos in Nuclear Systems ¹

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Abstract

The presence of chaos and quantum chaos is shown in two different nuclear systems. We analyze the chaotic behaviour of the classical SU(2) Yang–Mills–Higgs system, and then we study quantum chaos in the nuclear shell model calculating the spectral statistics of $A = 46\text{--}50$ atomic nuclei.

1. Introduction

A classical system is *chaotic* if has exponential divergence of initially closed trajectories in the phase space, i.e. small differences in the initial conditions produce great changes in the final behaviour. The phenomenon of chaos, which has become very popular, rejuvenated interest in nonlinear dynamics. Through numerical simulations on modern computers the presence of chaos has been discovered to be pervasive in many dynamical systems of physical interest [1].

In quantum mechanics one cannot apply classical concepts and methods directly being the notion of trajectory absent. Nevertheless, many efforts

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have been made to establish the features of quantum systems which reflect the qualitative difference in the behaviour of their classical counterparts [2].

The *quantum chaos*, or better the *quantum chaology* [3], studies the properties of quantum systems which are classically chaotic. The energy fluctuation properties of systems with underlying classical chaotic behaviour and time-reversal symmetry agree with the predictions of the Gaussian Orthogonal Ensemble (GOE) of random matrix theory, whereas quantum analogs of classically integrable systems display the characteristics of the Poisson statistics [2–4].

In this paper we analyze the presence of chaos and quantum chaos in two systems of interest for nuclear physics. In the first part we study analytically the suppression of classical chaos in the spatially homogenous SU(2) Yang–Mills–Higgs (YMH) system. In the second part we analyze numerically quantum chaos in the nuclear shell model by studying the spectral statistics of low-lying states of $A = 46\text{--}50$ atomic nuclei.

2. Chaos in the SU(2) Yang–Mills–Higgs system

In the last years there has been much interest in the chaotic behaviour of classical field theories [5]. Usually the order–chaos transition in these systems has been studied numerically by using Lyapunov exponents and Poincarè sections [1,2]. Less attention has been paid to analytical criteria [6,7]. Here we study the spatially homogenous SU(2) YMH system. Obviously, the constant field approximation implies that our SU(2) YMH system is a toy model for classical non-linear dynamics, with the attractive feature that the model emerges from particle physics.

The lagrangian density for the SU(2) YMH system is given by [8]:

$$L = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + \frac{1}{2}(D_\mu\phi)^+(D^\mu\phi) - V(\phi) , \quad (1)$$

where:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{abc} A_\mu^b A_\nu^c , \quad (2)$$

$$(D_\mu\phi) = \partial_\mu\phi - igA_\mu^b T^b\phi , \quad (3)$$

with $T^b = \sigma^b/2$, $b = 1, 2, 3$, generators of the SU(2) algebra, and where the potential of the scalar field (the Higgs field) is:

$$V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4 . \quad (4)$$

We work in the (2+1)-dimensional Minkowski space ($\mu = 0, 1, 2$) and choose spatially homogenous Yang–Mills and the Higgs fields:

$$\partial_i A_\mu^a = \partial_i \phi = 0, \quad i = 1, 2 \quad (5)$$

i.e. we consider the system in the region in which space fluctuations of fields are negligible compared to their time fluctuations.

In the gauge $A_0^a = 0$ and using the real triplet representation for the Higgs field we obtain:

$$\begin{aligned} L = & \frac{1}{2}(\dot{\vec{A}}_1^2 + \dot{\vec{A}}_2^2) + \dot{\vec{\phi}}^2 - g^2[\frac{1}{2}\vec{A}_1^2\vec{A}_2^2 - \frac{1}{2}(\vec{A}_1 \cdot \vec{A}_2)^2 + \\ & + (\vec{A}_1^2 + \vec{A}_2^2)\vec{\phi}^2 - (\vec{A}_1 \cdot \vec{\phi})^2 - (\vec{A}_2 \cdot \vec{\phi})^2] - V(\vec{\phi}), \end{aligned} \quad (6)$$

where $\vec{\phi} = (\phi^1, \phi^2, \phi^3)$, $\vec{A}_1 = (A_1^1, A_1^2, A_1^3)$, and $\vec{A}_2 = (A_2^1, A_2^2, A_2^3)$.

When $\mu^2 > 0$ the potential V has a minimum in $|\vec{\phi}| = 0$, but for $\mu^2 < 0$ the minimum is:

$$|\vec{\phi}_0| = \left(\frac{-\mu^2}{4\lambda}\right)^{\frac{1}{2}} = v$$

which is the non zero Higgs vacuum. This vacuum is degenerate and after spontaneous symmetry breaking the physical vacuum can be chosen $\vec{\phi}_0 = (0, 0, v)$. If $A_1^1 = q_1$, $A_2^2 = q_2$ and the other components of the Yang–Mills fields are zero. The system hamiltonian in the Higgs vacuum is:

$$H = \frac{1}{2}(p_1^2 + p_2^2) + g^2v^2(q_1^2 + q_2^2) + \frac{1}{2}g^2q_1^2q_2^2, \quad (7)$$

where $p_1 = \dot{q}_1$ and $p_2 = \dot{q}_2$. Obviously $w^2 = 2g^2v^2$ is the mass term of the Yang–Mills fields.

The transition order–chaos in systems with two degrees of freedom may be studied by the curvature criterion of potential energy [6]. It is however important to point out that *in general* the curvature criterion guarantees only a *local instability* and should therefore be combined with the Poincarè sections [9].

At low energy the motion near the minimum of the potential:

$$V(q_1, q_2) = g^2v^2(q_1^2 + q_2^2) + \frac{1}{2}g^2q_1^2q_2^2, \quad (8)$$

where the curvature is positive, is periodic or quasiperiodic and is separated from the instability region by a line of zero curvature; if the energy is increased, the system will be for some initial conditions in a region of negative curvature, where the motion is chaotic. According to this scenario, the energy of order→chaos transition E_c is equal to the minimum value of the line of zero gaussian curvature $K(q_1, q_2)$ on the potential-energy surface. For our potential the gaussian curvature vanishes at the points that satisfy the equation:

$$\frac{\partial^2 V}{\partial q_1^2} \frac{\partial^2 V}{\partial q_2^2} - \left(\frac{\partial^2 V}{\partial q_1 \partial q_2} \right)^2 = (2g^2 v^2 + g^2 q_2^2)(2g^2 v^2 + g^2 q_1^2) - 4g^4 q_1 q_2 = 0. \quad (9)$$

It is easy to show that the minimal energy on the zero-curvature line is given by:

$$E_c = V_{min}(K = 0, \bar{q}_1) = 6g^2 v^4, \quad (10)$$

and occurs at $\bar{q}_1 = \pm\sqrt{2}v$. Therefore the chaos-order transition depends on the parameter $\epsilon = g^2 v^4 / E$: for $0 < \epsilon < 6$ a relevant region of the phase-space is chaotic, while for $\epsilon > 6$ the system becomes regular. This result shows that the Higgs field value in the vacuum v plays an important role: for large values it makes the system regular in agreement with previous numerical calculations of Savvidy [5]. Also the Yang–Mills coupling constant g has the same role. Instead for fixed v and g there is an order–chaos transition increasing the energy E [10].

3. Quantum chaos in the nuclear shell model

One of the best systems for the study of quantum chaos is the atomic nucleus, which has been the subject of many investigations [11]. In atomic nuclei, the fluctuation properties of experimental energy levels are best studied in the domain of neutron and proton resonances near the nucleon emission threshold, where a large number of levels with the same spin and parity in the same nucleus are present, and an excellent agreement with GOE predictions has been found [12].

In this work we undertake the statistical analysis of the shell–model energy levels in the $A = 46$ –50 region. By using second–quantization notation, the nuclear shell–model hamiltonian may be written as [13]:

$$H = \sum_{\alpha} \epsilon_{\alpha} a_{\alpha}^+ a_{\alpha} + \sum_{\alpha\beta\gamma\delta} < \alpha\beta | V | \delta\gamma > a_{\alpha}^+ a_{\beta}^+ a_{\gamma} a_{\delta}, \quad (11)$$

where the labels denote the accessible single-particle states, ϵ_α is the corresponding single-particle energy, and $\langle \alpha\beta|V|\delta\gamma \rangle$ is the two-body matrix element of the nuclear residual interaction.

Exact calculations are performed with the $f_{7/2}$, $p_{3/2}$, $f_{5/2}$, and $p_{1/2}$ single-particle states, assuming a ^{40}Ca inert core. The diagonalizations are performed in the m -scheme using a fast implementation of the Lanczos algorithm with the code ANTOINE [14]. For a fixed number of valence protons and neutrons we calculate the energy spectrum for projected total angular momentum J and total isospin T . The interaction we use is a minimally modified Kuo-Brown realistic force with monopole improvements [15].

We calculate the $T = T_z$ states from $J = 0$ to $J = 9$ for all the combinations of 6 active nucleons, i.e. ^{46}V , ^{46}Ti , ^{46}Sc , and ^{46}Ca , and also for ^{48}Ca and ^{50}Ca .

Since we are looking for deviations from chaotic features, we are mainly interested in the low-lying levels up to a few MeV above the JT yrast line. For each JT set of levels the spectrum is mapped into unfolded levels with quasi-uniform level density by using the constant temperature formula [16].

The spectral statistic $P(s)$ is used to study the local fluctuations of the energy levels. $P(s)$ is the distribution of nearest-neighbour spacings $s_i = (\tilde{E}_{i+1} - \tilde{E}_i)$ of the unfolded levels \tilde{E}_i . It is obtained by accumulating the number of spacings that lie within the bin $(s, s + \Delta s)$ and then normalizing $P(s)$ to unity.

For quantum systems whose classical analogs are integrable, $P(s)$ is expected to follow the Poisson limit, i.e. $P(s) = \exp(-s)$. On the other hand, quantal analogs of chaotic systems exhibit the spectral properties of GOE with $P(s) = (\pi/2)s \exp(-\frac{\pi}{4}s^2)$ [2–4].

The distribution $P(s)$ is the best spectral statistic to analyze shorter series of energy levels and the intermediate regions between order and chaos. The $P(s)$ distribution can be compared to the Brody distribution [17]

$$P(s, \omega) = \alpha(\omega + 1)s^\omega \exp(-\alpha s^{\omega+1}), \quad (12)$$

with

$$\alpha = \left(\Gamma\left[\frac{\omega + 2}{\omega + 1}\right]\right)^{\omega+1}. \quad (13)$$

This distribution interpolates between the Poisson distribution ($\omega = 0$) of integrable systems and the GOE distribution ($\omega = 1$) of chaotic ones, and

thus the parameter ω can be used as a simple quantitative measure of the degree of chaoticity.

In order to obtain more meaningful statistics, $P(s)$ is calculated using the unfolded level spacings of the whole set of $J = 0\text{--}9$ levels for fixed T up to a given energy limit above the yrast line. Thus the number of spacings included is reasonably large.

Energy	^{46}V	^{46}Ti	^{46}Sc	^{46}Ca	^{48}Ca	^{50}Ca
≤ 4 MeV	1.14	0.90	0.81	0.41	0.58	0.67
≤ 5 MeV	1.10	0.81	0.96	0.53	0.58	0.69
≤ 6 MeV	0.93	0.94	0.99	0.51	0.66	0.62

Table 1: Brody parameter ω for the nearest neighbour level spacings distribution for $0 \leq J \leq 9$, $T = T_z$ states up to 4, 5 and 6 MeV above the yrast line in the analyzed nuclei.

Table 1 shows the best fit Brody parameter ω of the $P(s)$ distribution for the $J = 0\text{--}9$ set of level spacings in the $A = 46$ nuclei up to 4, 5 and 6 MeV above the yrast line. Clearly, ^{46}V , ^{46}Ti and ^{46}Sc are chaotic for these low energy levels, but there is a considerable deviation from GOE predictions in ^{46}Ca , which is a single closed-shell nucleus. In view of the peculiarity of this nucleus, we performed calculations for ^{48}Ca and ^{50}Ca , and obtained again strong deviations toward regularity, as the values of Table 1 show.

To explain these results we observe that the two-body matrix elements of the proton-neutron interaction are, on average, larger than those of the proton-proton and neutron-neutron interactions. Consequently the single-particle mean-field motion in nuclei with both protons and neutrons in the valence orbits suffers more disturbance and is thus more chaotic.

4. Conclusions

The presence of chaos in the homogenous $\text{SU}(2)$ Yang-Mills-Higgs system has been studied. The results show that for large values of the Higgs field in the vacuum there is a suppression of the chaotic behaviour of the Yang-Mills fields. In the future it will be of great interest to analyze this effect also in a non homogenous situation.

In the study of the nuclear shell model we have seen that the disturbance of single-particle motion by the two-body interaction is greatest in light nuclei, where the size of the range of the single-particle orbits is not much longer than the range of the nuclear force. In particular, for Ca isotopes, we find significant deviations from the predictions of the random-matrix theory which suggest that some spherical nuclei are not as chaotic in nature as the conventional view assumes.

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